4. $\sin\alpha - \sin(\alpha+\beta) + \sin(\alpha+2\beta) - \sin(\alpha+3\beta)$ ① + ---- to n terms.

Bsc part-I (ii) paper-I, Groups, summation of series

$LHS = \sin\alpha + \sin(\pi+\alpha+\beta) + \sin(2\pi+\alpha+2\beta)$ cosine $+ \sin(3\pi+\alpha+2\beta) + \cdots$ to n terms

positive sum of sines whose angles are in A.P formula [illegible]

$$= \frac{\sin n\cdot\frac{\pi+\beta}{2}}{\sin\frac{\pi+\beta}{2}}\sin\left\{\alpha+(n-1)\frac{\pi+\beta}{2}\right\} \text{ Ans}$$

5. $\sqrt{1+\cos2\alpha} - \sqrt{1+\cos4\alpha} + \sqrt{1+\cos6\alpha} - \cdots$ to n terms

$S = \sqrt{2\cos^2\alpha} - \sqrt{2\cos^2 2\alpha} + \sqrt{2\cos^2 3\alpha} - \cdots$

$= \sqrt{2}\cos\alpha - \sqrt{2}\cos2\alpha + \sqrt{2}\cos3\alpha - \cdots$

$= \sqrt{2}\left[\cos\alpha - \cos2\alpha + \cos3\alpha - \cos4\alpha + \cdots\right]$

$= \sqrt{2}\left[\cos\alpha + \cos(\pi+2\alpha) + \cos(2\pi+3\alpha) + \cos(3\pi+4\alpha) + \cdots\right]$

$$= \sqrt{2}\,\frac{\sin n\frac{\pi+\alpha}{2}}{\sin\frac{\pi+\alpha}{2}}\cos\left\{\alpha+(n-1)\frac{\pi+\alpha}{2}\right\} \text{ Ans}$$

6(i) $\sqrt{1+\sin\alpha} + \sqrt{1+\sin2\alpha} + \sqrt{1+\sin3\alpha} + \sqrt{1+\sin4\alpha} + \cdots$ to n terms

$= \sqrt{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} + \sqrt{\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha} + \sqrt{\cos^2\frac{3\alpha}{2} + \sin^2\frac{3\alpha}{2} + 2\sin\frac{3\alpha}{2}\cos\frac{3\alpha}{2}} + \cdots$

$= \sqrt{\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)^2} + \sqrt{(\cos\alpha + \sin\alpha)^2} + \sqrt{\left(\cos\frac{3\alpha}{2} + \sin\frac{3\alpha}{2}\right)^2} + \cdots$

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$$= \left(\cos\tfrac{\alpha}{2} + \sin\tfrac{\alpha}{2}\right) + \left(\cos\alpha + \sin\alpha\right) + \left(\cos\tfrac{3\alpha}{2} + \sin\tfrac{3\alpha}{2}\right) + \cdots$$

$$= \left\{\cos\tfrac{\alpha}{2} + \cos\alpha + \cos\tfrac{3\alpha}{2} + \cdots\right\} + \left\{\sin\tfrac{\alpha}{2} + \sin\alpha + \sin\tfrac{3\alpha}{2} + \cdots\right\}$$

$$= \frac{\sin n\frac{\alpha}{4}}{\sin\frac{\alpha}{4}} \cos\left(\tfrac{\alpha}{2} + (n-1)\tfrac{\alpha}{4}\right) + \frac{\sin n\frac{\alpha}{4}}{\sin\frac{\alpha}{4}} \sin\left\{\tfrac{\alpha}{2} + (n-1)\tfrac{\alpha}{4}\right\}$$

$$= \frac{\sin\frac{n\alpha}{4}}{\sin\frac{\alpha}{4}} \left\{\cos\frac{2\alpha + (n-1)\alpha}{4} + \sin\frac{2\alpha + (n-1)\alpha}{4}\right\}$$

$$= \sin\frac{n\alpha}{4}\,\mathrm{cosec}\,\frac{\alpha}{4} \left\{\cos\frac{2\alpha + n\alpha - \alpha}{4} + \sin\frac{2\alpha + n\alpha - \alpha}{4}\right\}$$

$$= \sin\frac{n\alpha}{4}\,\mathrm{cosec}\,\frac{\alpha}{4} \left\{\cos\frac{(n+1)\alpha}{4} + \sin\frac{(n+1)\alpha}{4}\right\}$$

6 (ii) ऊपर के तरह करेगा!

$$\tan^{-1}\frac{x-y}{1+xy} = \tan^{-1}x - \tan^{-1}y$$

7 (i) $\cos\theta\cos 2\theta + \cos 3\theta\cos 4\theta + \cos 5\theta\cos 6\theta + \cdots$ to n terms

दो का product है अगर हमेशा formula use नहीं होगा 2 से दोनों side में गुणा करना।

Let $S =$

$$\therefore 2S = 2\cos 2\theta\cos\theta + 2\cos 4\theta\cos 3\theta + 2\cos 6\theta\cos 5\theta + \cdots \text{ to } n \text{ terms}$$

$$= \cos(3\theta) + \cos\theta + \cos 7\theta + \cos\theta + \cos 11\theta + \cos\theta + \cdots \text{ to } n \text{ terms}$$

$$= \{\cos 3\theta + \cos 7\theta + \cos 11\theta + \cdots\} + \{\cos\theta + \cos\theta + \cos\theta + \cdots\}$$

$$= \frac{\sin n\frac{4\theta}{2}}{\sin\frac{4\theta}{2}} \cos\left\{3\theta + (n-1)\frac{4\theta}{2}\right\} + n\cos\theta$$

यहाँ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

or $2S = \dfrac{\sin n 2\theta}{\sin 2\theta} \cos\left\{3\theta + 2n\theta - 2\theta\right\} + n\cos\theta$ (3)

$= \dfrac{\sin n 2\theta}{\sin 2\theta} \cos\left\{2n\theta + \theta\right\} + n\cos\theta$

$\therefore S = \dfrac{1}{2}\left[\dfrac{\sin 2n\theta}{\sin 2\theta} \cos\left\{(2n+1)\theta\right\} + n\cos\theta\right]$ Ans

7(ii) ऊपर की तरह ही बनेगा।

8. prove $\dfrac{\sin\alpha + \sin 3\alpha + \sin 5\alpha + \dots \text{ to } n \text{ terms}}{\cos\alpha + \cos 3\alpha + \cos 5\alpha + \dots \text{ to } n \text{ terms}} = \tan n\alpha$

$N = \sin\alpha + \sin 3\alpha + \sin 5\alpha + \dots \text{ to } n \text{ terms}$

$= \dfrac{\sin n \frac{2\alpha}{2}}{\sin \frac{2\alpha}{2}} \sin\left(\alpha + (n-1)\frac{2\alpha}{2}\right\}$

$= \dfrac{\sin n\alpha}{\sin\alpha} \cdot \sin(\cancel{\alpha} + n\alpha - \cancel{\alpha})$

$= \dfrac{\sin n\alpha}{\sin\alpha} \cdot \sin n\alpha$ (1)

$D = \cos\alpha + \cos 3\alpha + \cos 5\alpha + \dots$

$= \dfrac{\sin n \frac{2\alpha}{2}}{\sin \frac{2\alpha}{2}} \cos\left\{\alpha + (n-1)\frac{2\alpha}{2}\right\}$

$= \dfrac{\sin n\alpha}{\sin\alpha} \cos\left\{\alpha + n\alpha - \alpha\right\}$

$= \dfrac{\sin n\alpha}{\sin\alpha} \cos\left\{n\alpha\right\}$ (2)

$\therefore L.H.S. = \dfrac{N}{D}$

$= \dfrac{\frac{\cancel{\sin n\alpha}}{\cancel{\sin\alpha}} \sin n\alpha}{\frac{\cancel{\sin n\alpha}}{\cancel{\sin\alpha}} \cos n\alpha}$

$= \frac{\sin x h}{\cos x h} = \tan x h$ proved.